Understanding Level of Certainty

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INTRODUCTION

After many years of teaching surveying measurement analysis, both to undergraduate students and practicing surveyors, I have observed that some people have difficulty understanding the concept of level of certainty regarding measurement errors. The purpose of this essay is to clarify this concept so that theoretically correct statements of measurement error can be made and applied, and that errors stated as being within certain levels of certainty can be interpreted correctly.

> *"A person who is 100% certain is, by definition, a fool."*

First, we must accept and recognize that errors exist in measurement. After fully accepting this reality, and recognizing some of the methods of estimating the errors, the next step is determining a level of certainty. In other words, if someone doesn't accept the reality and nature of errors in measurements, the rest of this will not make much sense.

The concept of level of certainty, sometimes called percent probability or level of confidence, affects our lives daily, whether we realize it or not. We have learned to process a statement by the "weather man" as to the percent chance of rain, for example, and decide whether or not to carry an umbrella. We see an opinion poll on television as to which candidate is most favoured by a sampling of the population, and begin to form opinions of our own. The cost of automobile insurance is affected by statistics of traffic accidents in the location where we live. Life insurance premiums and annuity payments are based on mortality tables, based on average life expectancy.

All of the above has something to do with statistics and probability, and sampling of data, with subsequent analysis of the data. The fact is, we live and operate in a world surrounded by countless statistical probabilities. Even if no statistical tests and quantification of data were ever made, the probabilities still exist.

Some like to quantify everything; others like to use "gut feelings" or experience to decide between alternatives. Either way, rational people make decisions, consciously or unconsciously, subjectively or scientifically, applying some form of "level of certainty". Suppose I am ordering dinner at a restaurant. I know from experience that I have not liked corned beef and cabbage in the past, but my mouth waters when I just think of filet mignon. To put some numbers on this, I may be unconsciously assigning a 5% probability that I will like the corned beef and cabbage (with 95% probability that I will not like it), and 90% probability that I will like the filet mignon (with 10% chance that I will not like it). Note that I did not go all the way with a 0% or 100% on either, since experience in living has taught me that maybe some cook just might be able to prepare corned beef and cabbage so that I can enjoy it - unlikely, but worth a 5% chance. Likewise, I know that some cooks cannot get "medium rare" right, or that the particular cut of beef will not be choice, so I might assume that 1 time out of 10, I will be disappointed with the steak.

A PHILOSOPHICAL EXPLANATION OF PROBABILITY

Well balanced, mature people generally have some doubt on most decisions. They realize that most decisions are made with incomplete or distorted information, and through personal filters of bias, fear, pride, prejudice, and individual experience. Often, we might feel confident about well-researched and considered judgments, but we discover later that those judgments were flawed. That should teach the fallacy of thinking that 100% certainty can exist (unless, of course you rationalize and blame someone else for the poor judgments).

A person who is 100% certain is, by definition, a fool. Such a person is closed minded, allowing no room for mind change based on added knowledge or consideration of new evidence or insights, facts which come to light, or persuasive debate. It is healthy to have self-confidence when judgments are well considered. But, some measure of caution and doubt is healthy too. You may feel 100% sure, in the midst of your pride and ego, but such level of confidence denies reality.

The real, hard, facts of life are that you or I can never, honestly, be 100% sure of anything (that which we choose to believe out of an act of faith being an exception). The proof of these statements will become evident when we discuss the concept based on mathematical science. When we see it from both a philosophical and a measurement science approach, reconciling the two as one reality, it all makes sense. Got your attention yet? Reflect on the above, and take a break if necessary. But, don't go away.

EVERYDAY EXAMPLES OF NUMERICAL PROBABILITY

If I put one white marble and one black marble in a box, what is the probability of reaching in and grabbing the white marble? Of course, it is 50%. If I play "russian roulette" with a six-shooter and one bullet, what are my chances of surviving the game? Assuming the bullet and gun are not defective and my head is not too hard, it is 5/6 or about 83%. What is the probability of drawing an "ace" from a full deck of cards? The answer is 1/13. Gamblers play constant games of probability when playing cards, dice, or the machines. Every reasonably intelligent person should know (intellectually, but perhaps not always emotionally), that we cannot win in the long run on the gambling tables or machines, since the probabilities are created in favour of the house. How did you think they got the money to build those casinos?

This last example, perhaps reveals why it may sometimes be difficult to teach level of certainty to some people. If a person denies that this variable called "probability" or "level of certainty" exists and must be reckoned with, that person is difficult to teach much about surveying measurement, or anything else where risk is involved. With that discouraging possibility ringing in my head, I choose to continue anyway. If you don't understand the concept, but have read past this point, maybe the effort will be worthwhile. You are teachable. And, maybe we'll save you some money at the gaming tables, too!

LEVEL OF CERTAINTY IN MEASUREMENT

What does all of this have to do with the world of measurement? It is this... any measurement has three values or numbers associated with it. That's right - not one, not two, but three. First is the estimate (all measurements are estimates) of the size or the quantity. Then, since it is an estimate of a continuous (another statistician's term) number, there must be some estimate of the range of error. This range of error is often called the "uncertainty", but more commonly simply the "error". It is an estimate of the precision of the measurement when investigating only the random errors, and an estimate of the accuracy when evaluating the extent that systematic errors have been discovered, quantified, and removed from the observations. In actuality, this estimate is some function of both precision and accuracy.

Now, we come to the "third dimension" to a measurement - level of certainty. We see it in National Map Accuracy Standards, where it is stated that "90% of the well defined horizontal positions shown on the map shall be within 1/50 inch", and "90% of the elevations interpolated from the map shall be within 1/2 the contour interval". This is a statement saying we are 90% sure of our accuracy, within stated limits. Only nine out of ten of the points tested would need to pass the test. One could fail and we would meet the standard.

The new GPS standards are on the 95% confidence level, as regards the relative positional error between adjacent points. The 95% confidence level seems to be emerging as the favoured level.

"...any measurement has three values or numbers associated with it."

AN EXAMPLE TO ILLUSTRATE

Here's how it works in statistical analysis. Once a sample of something is taken, the standard deviation of the readings can be computed. In measurement, the standard deviation is a measure of the precision of the method. If the normal probability curve is plotted, statistics people tell us that the area under the curve between plus and minus "sigma" (standard deviation) is 68.3% of the total area under the curve. This is not just some esoteric theory. It works. In making tests of the reading precision of 1" theodolites, I usually take 25 readings, estimating to 0.1" After calculating sigma, a count of the readings falling between plus and minus sigma (with respect to the mean) is nearly always exactly 17 (which is 68% of 25). I have never found it to be less than 16 nor more than 18. You can be off a little since the sample set is, after all, finite in size, not infinite.

Another way to understand the theory is to think of writing each of the 25 readings on a small piece of paper, and placing them in a box. What is the probability that one drawn at random will be between plus and minus sigma, with respect to the mean? The answer is 68%. The laws of randomness are as sound as other mathematical principles. The 68% probability here is as dependable as the 50/50 chance of drawing the white marble instead of the black marble, or a 25% chance of drawing a card in the suit of clubs rather than hearts, spades, or diamonds.

But, most of us would prefer not to work with only 25%, 50%, or even 68% certainty. Maybe you are Dwight D. Eisenhower and someone asked you how you felt about your decision to proceed with the Normandy Invasion and you said only 50%. Not too confident, huh? In reality, he probably had his doubts, but was more certain that 50% or 68%. Maybe even 90% or 95%. Was the man 100% sure? I doubt it. Ike was no fool.

In measurement, the way we determine certainty is much easier than when making those difficult decisions in life (such as invading Europe, dropping the bomb, getting married, etc.). Aside from statistics, how could you be more confident of an error statement? How about increasing the size of the error estimate? Wouldn't that work? Wouldn't I be more sure of my ability to pace a 100 foot distance within 10 feet than within 1 foot? It is illogical that the level of certainty would be the same for these two different error estimates. What would be the difference? Some people might "hmm-hah" around and just say "man, I'm sure of being within 10 feet but wouldn't bet my life on being within 1 foot." Well that's all right for the lay person, but not for a supposed expert in measurement, and rather quaint as a statement of error certainty on a survey plat. A "gut feeling" might be used, such as 99.9% sure of the 10 foot accuracy but only 50% sure of the 1 foot accuracy. But, "gut feeling" isn't good enough for professionals in the practice of measurement science, and not very defensible under the scrutiny of the prosecuting attorney who hired one of those college graduates who had three courses in this stuff.

LEVEL OF CERTAINTY USING STATISTICAL ANALYSIS

Finally, we come to the means to get it right - theoretically, scientifically, legally, morally, ethically, philosophically, without regard to sex, race, age, or political affiliation. This is it. Are you ready? Suppose I paced this distance 36 times and made an estimate each time (nearest 0.5 ft.) for the supposed 100 foot distance. The data is shown in a "frequency distribution table" below. "Frequency" means the number of times I observed a particular estimate.

A calculation for standard deviation yields 1.13 feet. This represents the 68.3% confidence level. I can declare to the world that my precision in pacing 100 feet, over similar terrain, same boots, etc. is 1.13 feet, but I am only

Reading	97.5	98.0	98.5	99.0	99.5	100.0	100.5	101.0	101.5	102.0
Frequency	1	2	3	4	6	8	4	4	4	2

68.3% confident of this statement. So, how do I gain more confidence? By increasing the error estimate! Statistics shows that, to be 90% sure, I multiply the 1.13 by 1.645. To be 95% sure, I double it. To be 99 % sure, I use 2.5 sigma. The "3-sigma" error is about 99.7% certainty. How do we have 100% certainty? Hopefully, you figured this out by now. You can't be 100% sure, unless the error is plus or minus infinity! Isn't that a wonderful insight to have? Remember, we said that anyone who is 100% sure is, by definition, a fool. Finite people cannot reach out to infinity. This fact, when fully digested, not only gives great insight to the realities of measurement, but also the realities of life - it keeps you humble and open-minded.

O.K., back to the theory of measurement. To continue my analysis, I can declare that I can pace a comparable distance to +/-1.9 feet and be 90% confident of this estimate, +/-2.3 feet and be 95% confident, +/-2.8 feet with 99% confidence, +/-3.4 feet with 99.7% confidence, etc. (Use the constants given to compute these precision indexes.) Isn't it logical, that to be more certain, you must widen the error range? It should make sense, logically. It also makes sense theoretically, and these are the accepted numbers to quantify it.

"...you must always evaluate the errors."

The above is repeatable. Another test, under similar conditions, should yield about the same standard deviation. All you need is about 20 or so minimum in your sample set for most tests. You can test nearly any measuring system this way. The standard deviation, as an analytical tool, is powerful. Its other applications are not the subject matter here, however. Hopefully, its use was valuable in demonstrating the concept of percent certainty.

A WORD ABOUT ACCURACY

The mean of the above set of pacings is 99.875 feet. If the distance had been accurately measured with a tape or electronic device as 100.00 feet, with an error of no more than a hundredth or two, I can see that I was not calibrated in my pacing by about 0.125 feet. That's almost negligible, considering the method, but nevertheless is a systematic error, and demonstrates the need for calibration. My confidence as demonstrated here relates only to repeatability or precision. If I want to make a statement of accuracy, I need to investigate the systematic errors, and add a random error (using error propagation theory - the subject of more essays) to the standard deviation based on the repeatability test, said error being an estimate of the random errors in the systematic errors themselves.

If you have followed this part, you are inching toward understanding the concept of positional error in measurement. But, let's not confuse the main point of this essay any further with more theories.

SUMMARY

Hopefully, this article will serve to direct the thinking of surveyors and their technicians concerning the "third dimension" of surveying measurement. If you are an instrument operator or data manipulator of any sort, you must always evaluate the errors. Pushing buttons and manipulating numbers is not professional surveying. Error control is, however, for surveyors are supposed to be the experts in land measuring, and errors are always present when we perform this function.

To fully understand errors and keep them in control, make defensible and theoretically correct statements about the errors, and apply concepts such as positional error, we must always recognize the three numbers associated with any measurement. It is incomplete to express any measurement without some statement of the range of error. But, even this statement is incomplete without stating the confidence or level of certainty attached to it.

The concepts described in this essay must be fully accepted, understood, and applied in the world of surveying and mapping. If we do otherwise, we are not the experts in measurement we pretend to be. The nature of measurement is one of dealing with errors and probabilities. All measurements are but estimates or professional opinions, with the uncertainties that accompany any opinion. The true nature of measurements cannot be ignored, denied, or corrupted, unless we are willing to give up any claim of being professional measurers.

We live and operate daily in a world of probabilities. We make judgments constantly (or should), based on some subjective or quantifiable percent probability of the outcome of the decisions. Our measurements are no exception. When we learn to live with, think by, and apply probability, we will be more successful in our daily decisions and more honest with ourselves and others, and maintain an accurate image as professional surveyors.

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